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Selection Principles and Pattern Formation in Fluid Mechanics and Nonlinear Shell Theory

Final Technical Report

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I. Summary.

The general purpose of much of the completed research was to investigate various symmetry breaking problems in fluid mechanics by the use of structure parameters and selection principles. Although all of the nonlinear problems studied under the grant involved systems of partial differential equations, many of these problems led to the study of a single nonlinear operator equation of the form

(1) $$F(w, \lambda, \gamma) = 0, \quad w \in H, \quad \lambda \in \mathbb{R}^1, \quad \gamma \in \mathbb{R}^1 .$$

Here H is a Hilbert space, F: $H \times \mathbb{R}^1 \times \mathbb{R}^1 \to H$ is a smooth mapping with $F(0, \lambda, \gamma) = 0$ for all real λ and γ, λ is typically a "load" parameter (e.g., a Reynolds number) and γ is typically a "structure" parameter. Instead of varying only the load parameter λ as is often done in the study of such equations, one of the main ideas used was to vary also the structure parameter γ in such a way that stable solutions of (1) were obtained. In this way one determines detailed stability results by making use of the structure of the model equations and the known physical parameters of the problem. Such a structure parameter method improves upon previous analytic methods using degree theory, catastrophe theory, singularity theory, amplitude equations, or numerical methods using supercomputers. The approach described has been carried out successfully, e.g., for Bénard-type convection problems, Taylor-like problems for "short" cylinders, rotating Couette-Poiseuille channel flows, and rotating plane Couette flows (see the list of publications in II).

The main focus of the research during the last two years of the grant was on wave theories of vortex breakdown in a tube. The approach again made use of structure parameters so that the approach was not necessarily restricted to weakly nonlinear waves as in previous investigations of vortex breakdown. A number of preliminary results for inviscid axisymmetric flows

were obtained and some of these were described in detail in previous Semiannual Progress Reports. Although considerable progress was made in the case of special models, the general problem of vortex breakdown was not "solved". Work will continue to determine, in particular, large amplitude transition solutions for vortex breakdown in a tube and the NASA Technical Officer for the grant will be informed by means of reprints and preprints of any future progress.

II. Publications under NAG 2-278.

1. "Degenerate critical points in convection problems", Algebraic and Differential Topology, Global Differential Geometry and Applications: Marston Morse Memorial Volume, Teubner-Verlag Publishers, Leipzig, 1984, 170-183, (with G. Knightly).

2. "A selection principle for Bénard-type convection", *Arch. Rational Mech. Anal.* 88 (1985), 163-193, (with G. Knightly).

3. "Stability of cellular convection", *Arch. Rational Mech. Anal.* 97 (1987), 271-297, (with G. Knightly).

4. Rotating Couette-Poiseuille channel flow, *Rocky Mountain J. Math.*, 1987, to appear, (with G. Knightly).

5. "Primary and secondary steady flows of the Taylor problem", *J. Diff. Equations*, 1987, to appear (40 pages).

6. Spiral states in rotating plane Couette flow", Preprint, University of Colorado, 1987, submitted for publication, (with G. Knightly).